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Technical Memorandum 79639

Two-Stream Theory of Spectral Reflectance of Snow

(NASA-TM-79639) TWO-STREAM THEORY OF
SPECTRAL REFLECTANCE OF SNOW (NASA) 21 p HC
A02/MF A01 CSCI 20F

N79-18721

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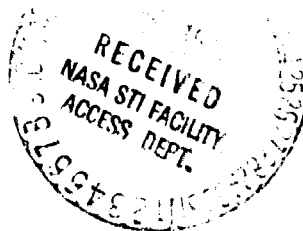
G3/74 17851

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SEPTEMBER 1978

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TWO-STREAM THEORY OF SPECTRAL REFLECTANCE OF SNOW

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ABSTRACT

Spectral reflectance of snow under diffuse illumination is studied using the two-stream approximation of the radiative transfer equation. The scattering and absorption within the snowcover due to the randomly distributed ice grains are characterized by the single scattering albedo and anisotropic phase function. Geometric optics calculations are used to relate the scattering and absorption parameters to the grain size and density of snow. Analytical expressions for the intensity within the snowpack and the asymptotic flux extinction coefficient are also obtained. Good agreement is shown between the theory and available experimental data on visible and near-infrared reflectance, and the asymptotic flux extinction coefficient. The theory also may be used to explain the observed effect of aging on the snow reflectance.

TWO-STREAM THEORY OF SPECTRAL REFLECTANCE OF SNOW

INTRODUCTION

Interest in developing techniques for remote measurement of snow parameters (e.g., density, grain size, water content) have increased significantly in recent years. One of the more promising techniques to monitor these parameters is that of microwave radiometry. A recent study by Chang and Choudhury (1978) has used a radiative transfer model to explain the observed brightness temperature of polar firm from the measured physical temperature, density and grain size. Analysis of spectral reflectance is an alternative, and complementary to microwave radiometry. Apart from remote sensing purposes, the study of spectral reflectance is relevant in climatology because it determines the amount of solar radiation absorbed by the earth's surface.

There have been several theoretical attempts to relate the spectral reflectance of a model snowcover with its physical properties (e.g., Dunkle and Bevens, 1956; Giddings and La Chapelle, 1961; Bergen, 1970, 1971, 1975; Bohren and Barkstrom, 1974). These theories either contain parameters which cannot be obtained directly from the measured physical properties or are valid for a restricted range within the visible spectrum.

For visible and near-infrared radiation, the scattering by individual ice grains is such that a major portion of the radiation gets scattered in the forward direction (Bohren and Barkstrom, 1974). The high reflectivity of snow is due to the multiple scattering effect within snow. Calculation of spectral reflectance necessarily involves a study of radiative transfer in snow. For this purpose, the radiative transfer equation is useful if snow is represented by randomly distributed, well separated ice grains. The snow parameters enter into the radiative

transfer equation through the single scattering albedo, the extinction coefficient and the phase function.

The physical properties of a natural snowcover are generally not known accurately and are, at best, approximately homogeneous. Although an accurate solution of the radiative transfer equation will be useful for standard reference and comparison purposes, to date no such calculation has been performed. Based on an approximate solution of the radiative transfer equation, a simplistic theory of spectral reflectance is given in this paper. All parameters in the theory can be calculated directly from the measured physical properties of snow, and the theory is applicable to visible and near-infrared radiation. The exact points of departure between the analysis presented below and the previous reported results vary. Instead of discussing these differences, a comparison will be made to show the relationship among the results. In addition, comparisons will be made between the theory and the observations.

APPROXIMATE EXPRESSION FOR REFLECTANCE

To obtain a closed form analytic expression for reflectance, the following assumptions will be made:

1. The snowcover is homogeneous.
2. Radiative transfer within the snowcover can be studied by dividing the radiation field into two streams of intensities.
3. Geometric optics calculations can be used to obtain the scattering and absorption parameters of individual ice grains.

With the first assumption the presence of internal inhomogeneities is neglected. This assumption is particularly crucial for a natural snowpack because of metamorphism. This is also a common assumption in all previous models.

The two-stream representation of the radiation field is a widely used method, and shown to be a reasonably accurate approximation of the radiative transfer equation (e.g., Domoto and Wang, 1974; Coakley and Chylek, 1975; Wiscombe and Grams, 1976; Venkatram and Viskanta, 1977). Although multiple scattering problems can be solved analytically with this approximation, the use of this approximation is a weakness of this study. A special form of this approximation was used previously in the calculation of snow reflectance (Dunkle and Bevans, 1956; Bergen, 1975).

In general form, the two-stream approximation corresponds to replacing the radiative transfer equation by the following coupled differential equations for mean intensities ($I_+(\tau)$ and $I_-(\tau)$) in the forward and the backward hemispheres:

$$\delta \frac{dI_+}{d\tau} = -I_+ + \omega(1 - \beta)I_+ + \omega\beta I_-; \quad \delta \frac{dI_-}{d\tau} = -I_- + \omega(1 - \beta)I_- + \omega\beta I_+$$

where ω is the single scattering albedo, and τ is the optical depth within the snow, given by

$$\tau = \gamma_e z$$

where γ_e is the extinction coefficient, and z is the depth measured from the snow surface within the snowcover. The coefficients δ and β are parameters, the choice of which distinguishes alternate forms of the two-stream approximation. Some of the choices of δ and β are as follows:

(i) Generalized Schuster-Schwartzschild (Wiscombe and Grams, 1976)

$$\delta_1 = \frac{1}{2}$$

$$\beta_1 = \frac{1}{2} \int_0^1 d\mu \int_0^1 d\mu' p(\mu, -\mu')$$

(ii) Modified Schuster-Schwartzschild (Sagan and Pollack, 1967; Lyzenga, 1973)

$$\delta_2 = \frac{1}{\sqrt{3}}$$

$$\beta_2 = \frac{1}{2} \left[1 - \frac{1}{2} \int_0^\pi p(\cos \theta) \cos \theta \sin \theta d\theta \right]$$

where $p(\mu, \mu')$ and $p(\cos \theta)$ have usual meaning of the phase function (Chandrasekhar, 1960).

It is expected that different set of δ and β will give different numerical results. The merit of a choice for a particular application should be decided by comparison with the exact solution of the radiative transfer equation. Unfortunately, as stated in the introduction, such a comparison cannot be made for the present application. In the following, the method of determining the parameters and the solution of the equations will be discussed.

The phase function needed for the calculation of β can be obtained from experimental observations or from a scattering theory calculation. Since there are no available experimental observations, one needs to perform a theoretical calculation by specifying the shape, size and the refractive index of ice grains. By discussing the difficulty and the limitations associated with the choice of shape, a recent study on snow (Bohren and Barkstrom, 1974) considered spherical non-absorbing particles and used geometric optics to calculate the phase function. Geometric optics calculations for transparent spheres (Bohren and Barkstrom, 1974; Hansen and Travis, 1974) give the value of the parameter β_2 as 0.065.

Using the approximate relationship between the parameters β_1 and β_2 (Wiscombe and Grams, 1976) the value of the parameter β_1 is obtained as $0.12 \pm .05$. The uncertainty in β_2 is due to the stated accuracy of the approximate relationship. Because of this uncertainty, only the modified two-stream approximation will be compared with observations. The single

scattering albedo, ω , can be obtained from the same calculational method which gives the phase function. A simple formula which reproduces the single scattering albedo with considerable accuracy as compared with the Mie theory results is (Irvine and Pollack, 1968):

$$\omega = \frac{1}{2} + \frac{1}{2} \exp(-1.67 k_{\lambda} r)$$

where k_{λ} is the absorption coefficient of ice (Hobbs, 1974; Irvine and Pollack, 1968), and r is the radius of the sphere. For non-spherical particles, one can use this formula by modifying the numerical coefficient in the exponential (Sagan and Pollack, 1967).

The extinction coefficient of snow γ_e is the product of the extinction cross-section and the number density of ice grains. By taking the extinction cross-section as $2\pi r^2$, one obtains

$$\gamma_e = \left(\frac{3}{2r}\right) \left(\frac{\rho}{\rho_i}\right)$$

where ρ and ρ_i are respectively the density of snow and ice.

All parameters in the radiative transfer equation have now been related with the physical properties of the snowcover.

If the surface of the snowpack is not ice-glazed, then for diffuse incident radiation, I_0 , the boundary conditions for the solution of the equations are:

$$I_+(0) = I_0$$

$$I_-(\tau_0) = R I_+(\tau_0)$$

where R is the reflectivity of the underlying surface and τ_0 is the total optical thickness of the snowcover,

$$\tau_0 = \gamma_e h.$$

where h is depth of the snowcover

Following standard procedures (Sobolev, 1960) the solution can be written as:

$$I_+(\tau) = \frac{C_1}{2} \left[1 + \frac{1-\omega}{\delta a} \right] e^{-a\tau} + \frac{C_2}{2} \left[1 - \frac{1-\omega}{\delta a} \right] e^{a\tau}$$

$$I_-(\tau) = \frac{C_1}{2} \left[1 - \frac{1-\omega}{\delta a} \right] e^{-a\tau} + \frac{C_2}{2} \left[1 + \frac{1-\omega}{\delta a} \right] e^{a\tau}$$

where

$$a^2 = \frac{(1-\omega)(1-\omega+2\omega\beta)}{\delta^2}$$

$$C_1 = \frac{2I_0}{\left[1 + \frac{1-\omega}{\delta a} \right] \left[1 + \frac{\alpha(\alpha-R)}{R\alpha-1} e^{-2a\tau_0} \right]}$$

$$C_2 = \frac{\left(\frac{\alpha-R}{R\alpha-1} \right) 2I_0 e^{-2a\tau_0}}{\left[1 + \frac{1-\omega}{\delta a} \right] \left[1 + \frac{\alpha(\alpha-R)}{R\alpha-1} e^{-2a\tau_0} \right]}$$

$$\alpha = 1 - \frac{2(1-\omega)^{1/2}}{(1-\omega+2\omega\beta)^{1/2} + (1-\omega)^{1/2}}$$

From this solution, the reflectance $A(\tau_0)$ can be calculated as:

$$A(\tau_0) = \frac{I_-(0)}{I_0}$$

$$= \frac{(R\alpha - 1)\alpha + (\alpha - R)e^{-2\alpha\tau_0}}{(R\alpha - 1) + \alpha(\alpha - R)e^{-2\alpha\tau_0}}$$

For a deep snowpack ($\tau_0 \rightarrow \infty$), the reflectance is given by

$$A(\infty) = \alpha$$

The asymptotic flux extinction coefficient is obtained as:

$$F = -\gamma_e \left[\frac{\left(\frac{dI_+(\tau)}{d\tau} \right)}{I_+(\tau)} \right]_{\tau=\infty}$$

$$= \left(\frac{3\rho}{2r\delta\rho_i} \right) [(1 - \omega)(1 - \omega + 2\omega\beta)]^{1/2}$$

COMPARISON WITH PREVIOUS CALCULATIONS AND OBSERVATIONS

The reflectance of a deep snowcover and the asymptotic flux extinction coefficient can be calculated from the grain size and the density of the snowcover. Besides the assumptions made in obtaining the analytical results, the model of spherical particles was used to calculate the optical properties of individual ice grains. Although the back-scattered fraction, β , is shown not very sensitive to the particle geometry (Wiscombe and Grams, 1976), other optical parameters (i.e., the single scattering albedo and the extinction coefficient) do depend upon the geometry. The ice grains of a snowcover are not spherical. The shape is generally oblong and of variable dimensions. Although average optical properties of the grains, due to their random orientation with respect to the mean intensity, may correspond to a sphere, reservation should be exercised regarding the applicability of the theory to a real snowpack. Thus, the comparison between the theory and the experimental observation presented below should not be regarded as conclusive.

Bergen (1970, 1975) related semi-empirically the snowpack parameters to the reflectance, and to the asymptotic flux extinction coefficient derived by Dunkle and Bevens (1956). Using the approximate solution of the radiative transfer equation derived by van de Hulst (1970, a, b), and taking the model of spherical ice grains, Bohren and Barkstrom (1974) related the snow parameters to the reflectance, and to the asymptotic flux extinction coefficient for visible radiation. Since Bohren and Barkstrom have compared their theory with Bergen's results, the following comparison will be restricted to Bohren and Barkstrom's theory. The status of the experimental observations of the reflectance and the flux extinction coefficient is disappointing either because of quantitative disagreement (Lilljequist, 1956; Thomas, 1963) or due to incomplete specification of snow parameters (O'Brien and Munis, 1975). Since Bohren and Barkstrom have compared with Lilljequist's observations, this comparison will be discussed first with the present theory. Figure 1 and Table 1 show this comparison for the asymptotic flux extinction, and the reflectance of a deep snow-cover. Although present theory appears to be little inferior to Bohren and Barkstrom's theory, it is difficult to vindicate either theory because the observations have not yet been duplicated. The discrepancy between the models is particularly noticeable in the asymptotic flux extinction coefficient. Whereas Bohren and Barkstrom predict lower values, the present theory predicts higher ones than the observations.

Figure 2 we show the calculated snow reflectance compared with the experimental values for a nearly fresh snow (O'Brien and Munis, 1975). The results of the calculation are in good agreement with the observations. All prominent spectral structures appearing in the observation are well duplicated in the calculations. The quantitative agreement is

also good but should be treated with caution because the experimental results are relative to a standard (white Barium Sulfate) reflector. It is however reassuring that the radius values used in the calculation are in the range expected for a fresh fallen snow.

The ice crystals of fresh fallen snow usually are of complex form and contain sharp corners. The shape of these crystals changes with time depending upon the vapor content and the prevalent temperature. The process of equi-temperature metamorphism leads to the production of fairly uniform and well rounded grains. At the initial stage of this metamorphism the mean radius of the ice crystals is about 0.2 mm. The radius then continues to increase as the process of metamorphism advances. At a fairly advanced stage of this metamorphism, the radius increases to about 1.0 mm. The effect of other metamorphism such as the temperature-gradient metamorphism or the melt-freeze metamorphism, is generally to produce non-spherical and non-uniform ice crystals also the crystals are larger than equi-temperature metamorphism. Thus, although it is not unique, one should be able to use the radius of ice crystal to characterize the stage of metamorphism. Figure 3 illustrates the changes in the calculated snow reflectance due to the difference in the crystal radius. The chosen radius are the typical snow crystal sizes for different stages of the equi-temperature metamorphism. The overall shape of the curve does not seem to depend upon the radius of the ice crystals but the relative magnitudes of various parts of the curve vary with the radius. The spectral reflectance in the red and near-infrared regions shows maximum sensitivity to the crystal sizes.

CONCLUSION

Based on the two-stream approximation of the radiative transfer equations, analytic results for the reflectance and the asymptotic flux extinction coefficient of a homogeneous snow-pack are derived. Using geometric optics calculations for spherical particles, the reflectance and the extinction coefficient are related to directly measurable snow parameters (density,

and the grain size). Good agreement was shown with the observed spectral dependence of the reflectance and the extinction coefficient using the measured (expected) snow parameters. Further observations are highly desirable to test the accuracy and hence to refine the calculation.

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Table 1

Comparison of Observed and Calculated Values of the Reflectance
Snow Parameters — $\rho = 0.43 \text{ gm/cm}^3$, $r = 0.15 \text{ mm}$)

Wavelength (μ)	Modified two-stream	Observation*	Bohren and Barkstrom
0.420	0.987	0.960	0.978
0.520	0.980	0.972	0.967
0.595	0.974	0.960	0.954
0.650	0.964	0.921	0.940

*Data, Lilljequist (1956).

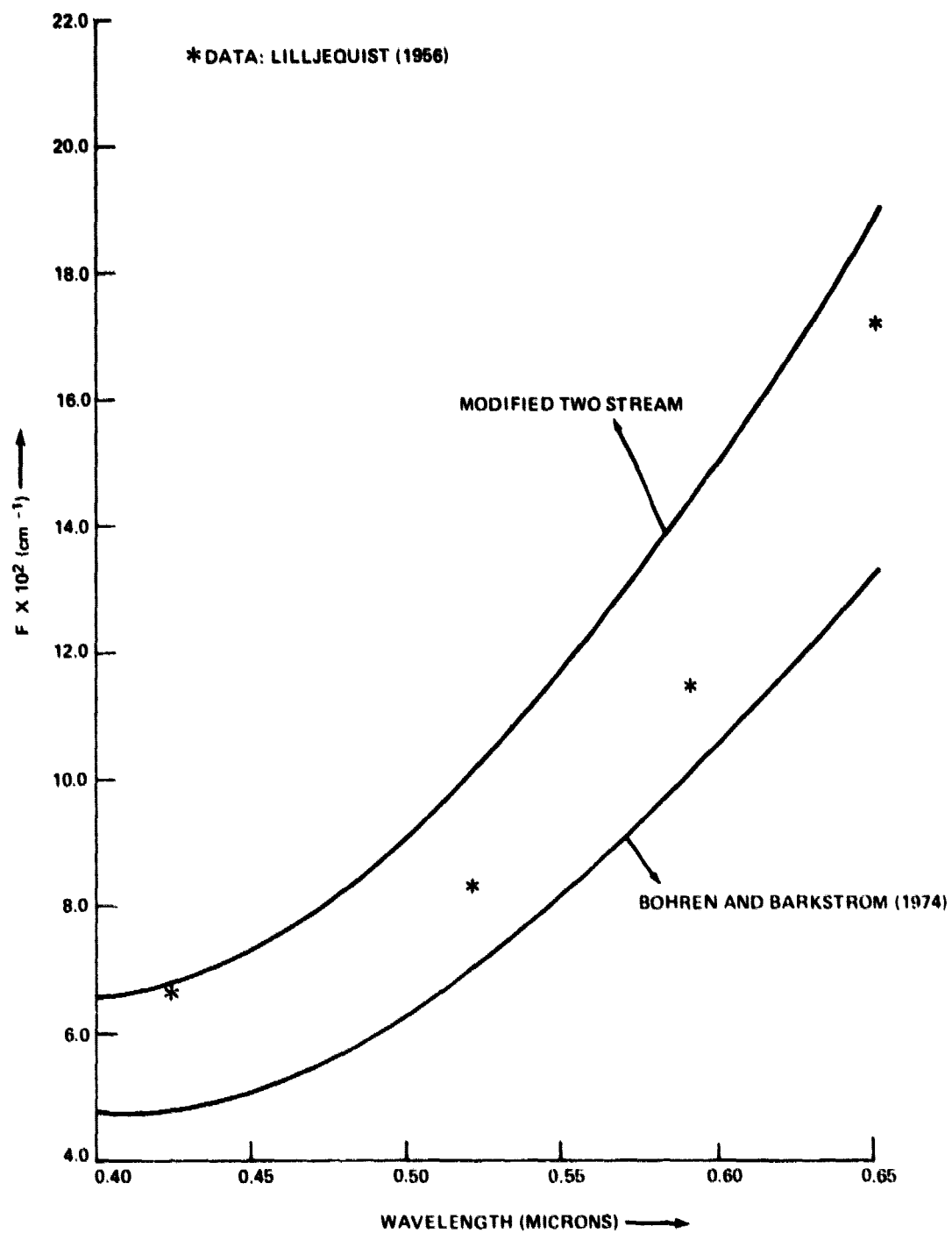


Figure 1. Comparison of Observed and Calculated Values of the Asymptotic Flux Extinction Coefficient.

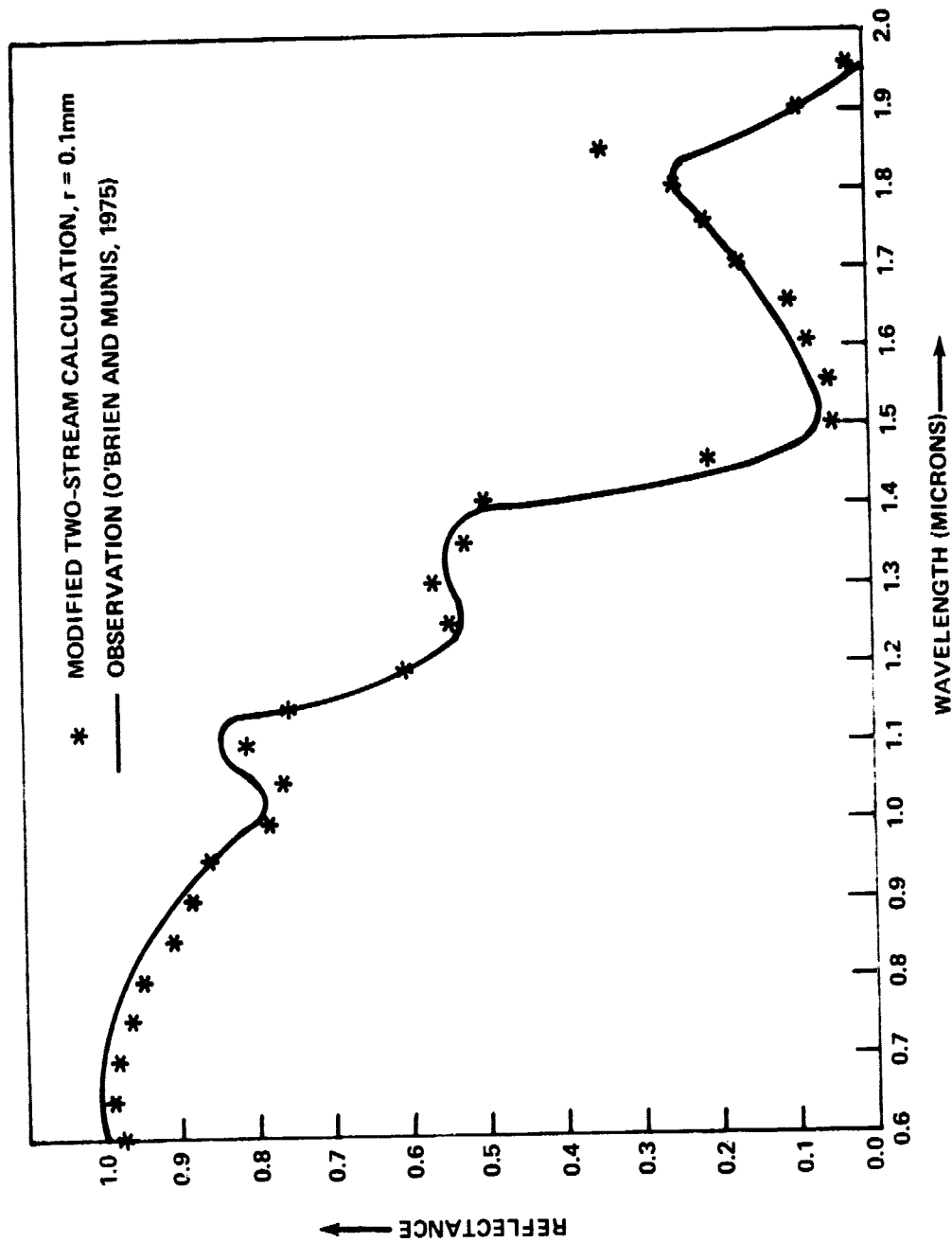


Figure 2. Comparison of Observed and Calculated Visible and Near Infrared Reflectance of Nearly Fresh Snowpack.

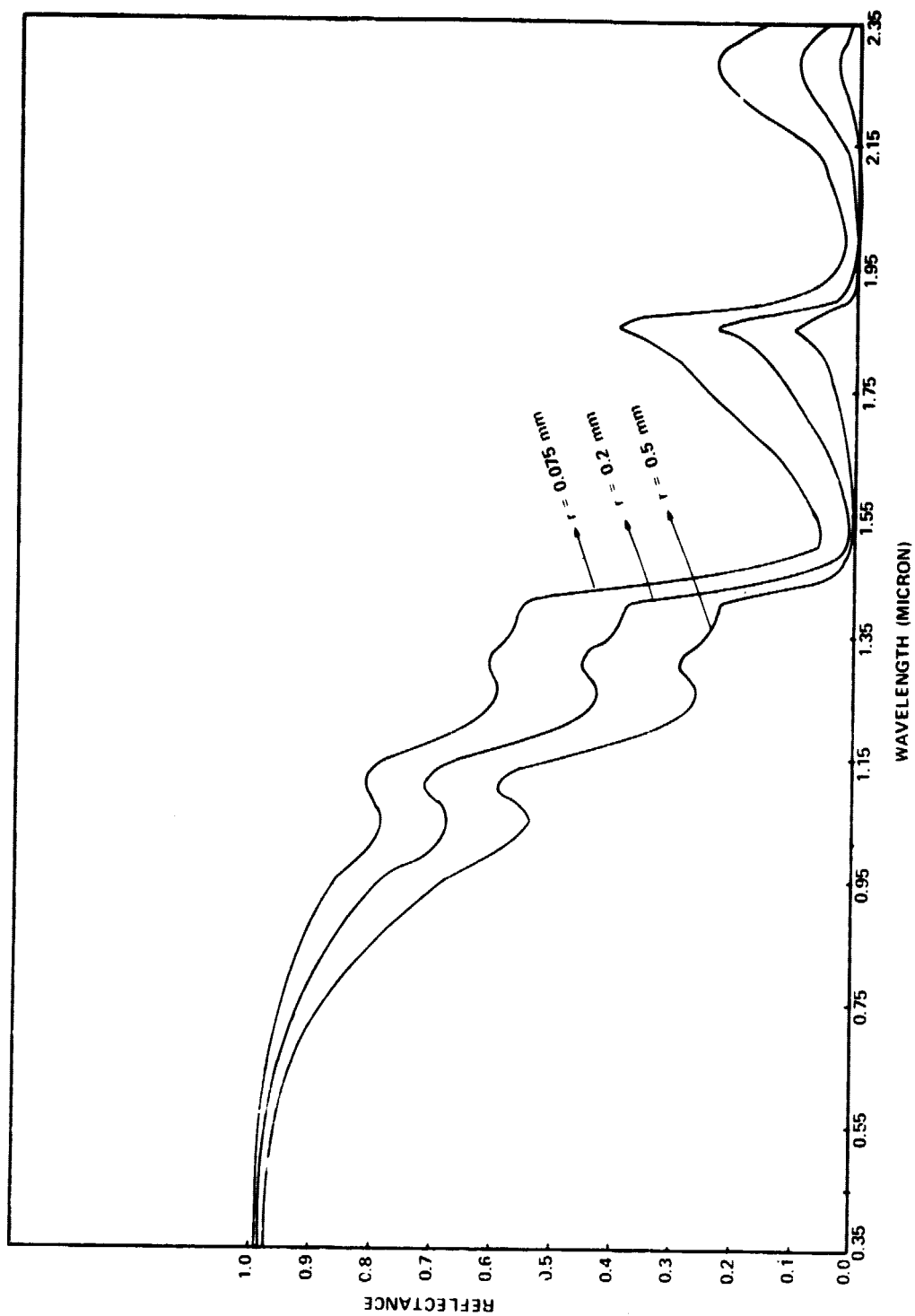


Figure 3. Illustration of the Effect of Different Snow Crystal on Snow Reflectance.